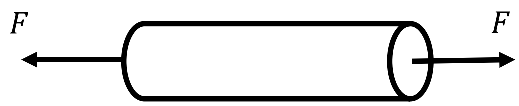
A random force $F \sim N(2{,}000, 100^2)$ N acts on the shaft as shown. The stress of the shaft is calculated by $S = F/A$, where A is the cross sectional area. If the maximum stress is $S_y = 13$ MPa, determine the minimum radius of the shaft so that the probability that the shaft will not fail is at least 0.9999. Hint: 1 Mpa = 10^6 Pa, and 1 Pa $= 1\,\frac{\mathrm{N}}{\mathrm{m}^2}$.



Solution

If the stress is less than the maximum stress, the shaft will not fail. Then the probability of no failure, given by the following equation, should be greater than or equal to 0.9999.

$$\Pr(S < S_y) = \Pr\left(\frac{F}{A} < S_y\right) = \Pr\left(\frac{F}{\pi r^2} < S_y\right) = \Pr(F < \pi r^2 S_y) = \Phi\left(\frac{\pi r^2 S_y - \mu_F}{\sigma_F}\right) \geq 0.9999$$

$$\frac{\pi r^2 S_y - \mu_F}{\sigma_F} \geq \Phi^{-1}(0.9999)$$

$$r \geq \sqrt{\frac{\sigma_F \Phi^{-1}(0.9999) + \mu_F}{\pi s_y}} = \sqrt{\frac{100(3.719) + 2000}{13(10^6)\pi}} = 7.62 \times 10^{-3}\ m$$

Therefore, we can design the shaft with a radius of 8 mm.